THE MATHEMATICS TEACHER

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RECENT ADVANCES IN THE TEACHING OF MATHEMATICS.

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In attempting a discussion of our subject, we are confronted by three possible lines of attack: First, what advances, if any, in the subject matter that is presented in the ordinary courses in mathematics; second, what improvements are to be noted in the methods of presentation of mathematical subjects to the classes; and third, what advancement is worthy of note among teachers of mathematics as to their professional training and fitness to be recognized as leaders in their chosen profession. Any one of these lines of thought is capable of extended discussion which exceeds the limits of this paper. We shall, therefore, set forth under each some points which appeal to us as worthy of presentation on a subject of such vital interest to us all.

As to the subject matter of mathematics, we may say that there has been but little, if anything at all, put forth in recent years, that can be classed as entirely new. A writer on geometry asserts that there has been nothing new in that subject for the past two thousand years; that we today are following practically the same line of development as did Euclid. But be that as it may, that there has been advancement is evidenced by the fact that emphasis is placed upon certain topics that are more important and vital in their relation to the subject of mathematics as a whole, and to the sciences that depend upon mathematics for their mastery. Emphasis is now placed on those

things that are in the main essential, practical, and fundamental to the industries and to the sciences, to the exclusion of those subjects that are unessential and impractical. There is, therefore, no longer use for such topics as circulating decimals, alligation, cube root, continued fractions, and others that might be mentioned, because the pupil's time can be better spent on subjects more useful to him.

We mention first that emphasis is now being placed on the complete mastery of the equation as the great instrument for use in all succeeding mathematical work. If a pupil is to be well prepared for successful work in the extensive fields of engineering, he must be able to handle equations rapidly and efficiently. He must take his equations in their complex forms as given, simplify them, reduce them, transform them when necessary, in order to find the values of the unknowns. In fact the whole subject matter of the algebra centers around the handling of the equation. We add, subtract, multiply, divide, factor, simplify fractions, find powers and roots, and learn other important processes, not so much for their value as a knowledge of how to do them, but rather that we may be able to deal effectively with equations of all kinds, whether simple, quadratic, or of higher power. And we as teachers fall short of doing our full duty if we fail to place proper emphasis on a complete mastery of the equation.

The subject of determinants calls for brief notice because of its value to all who extend their reading and study of mathematics beyond the elementary fields, and because of their great value in the solution of equations containing four, five, six, or even more unknown quantities. We place emphasis upon a knowledge of the determinant notation because of its relation to the higher mathematics, and because of its power and utility as an instrument in research work.

Another advancement is the correlation of algebra and physics. This has arisen from the fact that science teachers have complained that their pupils could not handle physical formulæ because they did not know their algebra. In reply, the teachers of mathematics asserted that their work had been well done, and that the fault lay not with the pupils, but with the science teachers in the presentation of their subject. In truth,

each was partly in the right and both were in the wrong. The pupils had been taught to solve for x, or y, or z, as the case might be, and had thus fallen into a certain habit of thinking algebraically. When physical problems came to be considered, it presented something entirely new to their way of thinking and consequently confusion naturally arose. To overcome these difficulties, our textbooks now contain a few pages of physical formulæ, such as $s = \frac{1}{2}gt^2$, $C = E/(R - r)$, $K = \frac{1}{2}(mv^2/g)$, etc., and problems relating to them. By solving for the unknown value and then making the required substitutions, the pupil gets away from the routine way of doing everything in x, y, or z. He is led to see that algebra is not only an end in itself, but also a means to successful work in physics, chemistry, and in the broader field of mechanics.

A third advancement in recent years is the introduction into elementary algebra of the subject of graphs, including the plotting of equations, the graphic solution of equations, and graphic analysis. Coming as it does from the analytic geometry, it was introduced into the elementary algebra as a preliminary step to that important subject. But its real value was soon appreciated as a separate subject even to those who might not pursue their mathematics further, because it enabled the pupils to have a much broader comprehension of the meanings attached to x and y. Abstract as much of our mathematical reasoning is, it can be made very concrete, and made to appeal strongly to the intellect, if we can visualize the conditions of a problem. For that which can be presented to the eye will always make a deeper and more lasting impression on the mind. We have all read a great deal of the events that have happened in Europe during the past eighteen months. But how eagerly do we look at a picture that portrays to us conditions as they really exist. By reading we get a general idea from which we build up our own individual mental images. But the picture gives a far more vivid impression. So in our teaching, mathematical relations may be discovered, or a truth driven home more forcibly, when by the use of the crayon or pencil a few lines are drawn, setting forth the conditions as they exist in some particular problem. Our pupils can solve rapidly such simple equations as $x + y = 3$ and $x - y = 5$, and get the values of x and y, but

surely they will get a much broader conception of what we mean by simultaneous equations when they learn that the values of x and y are the co-ordinates of the point of intersection of the graphs of these two equations. In solving the equations of higher degree, the results can be more easily determined and better understood, if they know that roots of the equation are the points of intersection of the graph of the equation with the x-axis or the y-axis, as the case might be. The use of graphs is also useful and practical in many other ways. The statistician uses them to present his facts and figures to the eye in a more telling manner; the broker and the merchant use them to record the rise and fall of prices; the physician to record the progress of disease; the corporation to compare its work from year to year. So we see that graphs have won their place not only in a well-balanced course in mathematics, but in the industrial world as well.

Another subject in which rapid advancement has been made in recent years is trigonometry, and we note a decided improvement not only in the manner in which the subject is developed and presented but also in the devices by which trigonometric formulæ may be derived and remembered for future use. The greatest advancement has been in the matter of textbooks on this subject. A textbook of 25 to 30 years ago contains but little more than the simple facts of the subject presented in a dry uninteresting manner. Recent textbooks present the subject in a more interesting and attractive manner, and develop the subject in a more effective way. As to devices which add to the value of the subject as aids in using the formulæ more readily and in remembering them, we mention one or two. In dealing with the functions of an angle, it is often necessary to pass from one function to another and to do it quickly. As, for example, having given the tan a/b, and we wish to know the sine, cosine, or any other function, for ready substitution. This can be obtained very quickly by actually drawing a right triangle, placing a as the side opposite to the given angle, and b as the side adjacent to the given angle, and then solving for the undetermined hypotenuse by the Pythagorean theorem. It is then possible to obtain any function of the angle at once and be sure you are right, without having to make use of the ordi-

nary transformations. This device is especially valuable in the handling of equations involving the inverse functions. We mention, also, a method by which it is fairly easy to remember the formulæ for the sum of the sines, for the difference of the sines, for the sum of the cosines, and for the difference of the cosines of two given angles. It is based on the well-known formulæ, $\sin (x+y) = \sin x \cos y + \cos x \sin y$, and $\cos (x+y) = \cos x \cos y - \sin x \sin y$, and in the order given. We recall that the coefficient in each case is 2, and that each formula involves one half the sum and one half the difference of the two angles. It remains for us to use the terms sin and cos in their proper places. The first term gives us sin and cos for the sum of the sines; the second, cos and sin for the difference of the sines; the third, cos and cos for the sum of the cosines; and the fourth, sin and sin for the difference of the cosines, with also the — sign in its proper place.

Under the subject of *method,* the greatest advancement in recent years is to be noted in the adoption of the so-called laboratory method, a term which is applied to any and all proposals that have for their underlying principle the one thought of adaptability and interest. The success or failure of any method is determined by whether or not it can be used to rouse the interest and hold the attention of the child. The laboratory method is good then, as judged by this standard, whether applied to the subject matter that we teach to our pupils, or the manner in which it is presented. By drawing upon subjects either in algebra or arithmetic with which our pupils are in a measure familiar, a lively interest is awakened, and they are ready to solve problems that arise out of their own experience, or are taken from topics about which they are accustomed to think. This means, then, that the problems should be real problems, drawn from fields of actual experience, in contrast to problems that are purely artificial or appeal only because they are humorous or ludicrous. We do not censure high-school pupils who do not become interested in many of the old "chestnuts" that are the heritage of the past, or problems such as this which are worthy of nothing more than a smile: "A woman walks 60 miles in 17 hours, walking 3 miles an hour uphill, and 4 miles an hour downhill. How many miles does she walk uphill and how many downhill?"

The sources of real problems are limited only by the pupil's individual interests. The boy on the farm will see in his arithmetic and algebra a means to a very important end if the work he is required to do is decidedly agricultural in its selection. The city boy will be all eyes and ears for his algebra if his teacher can make it plain to him that the large bridge over which he passes so often first existed in some one's mind, and that the same x and y, so often puzzling to him, had played their part in determining the stresses and stains, and the peculiar part each separate piece of iron and steel would have to endure in the completed structure. Our girls will not be continually asking the question, "What is the use of the study of algebra anyway?" if in our presentation of the subject we can cut loose from time-worn problems, and draw somewhat upon subjects that have a vital interest to girls in particular.

We are ready to take a more advanced position than this, based upon observation and study of conditions as we find them coming under our experience. There are certain facts to which we all can readily give assent. First, that our pupils do not all have the same mathematical ability; second, that all our pupils are not preparing for the same pursuits in life, and hence do not need the same preparation; third, that very many of our pupils under the best training possible to give them will be prepared to fill only mediocre positions at best, hence their training should be along lines that will better fit them for the greatest measure of success. Therefore, we are ready to assert that algebra as now required by commercial students is largely a waste of time from the standpoint of utility. We fully realize that algebra has its place for cultural value and mental training. And we would not lower in the least the amount of work required to take its place, but rather substitute in its stead a rigid course in commercial or industrial arithmetic, and our pupils will get the same amount of training and at the same time be better fitted for the commercial and industrial world. If we do not argue for such a change as this, how can we answer the challenge of all those pupils who leave our public schools and seek the business training in the private commercial school. Some one will say: Will not such a plan prepare our boys and girls for only one thing, and thus make them the victims of class education which will differentiate between them and those capable of greater

educational possibilities? But would it not be far better for our schools to give the best training possible to those whose capabilities are more limited than others, so as to fit them for a larger measure of success? The laboratory method, then, will not only aid us in determining largely how we shall teach, but also what we shall teach, so as to make mathematics a stepping stone to larger usefulness, rather than a stumbling block in the way of intellectual attainments.

But perhaps the best advancement of all, and the one which has been largely responsible for all the others we have named, is the improvement in the personnel of the teachers of mathematics themselves. And what we say on this point is applicable to the other members of the teaching profession as well. Teaching is no longer a step to other professions, because school boards have made it possible for those who engage in it to consecrate themselves wholly to this work by making the remuneration somewhat in keeping with the measure of service required. We have, therefore, met together as a body of teachers, not so much to hear papers read and discussed, as to exchange views each with the other, and find out what others are doing in the same lines of work as we ourselves are engaged. By doing so, we keep from getting into a rut and following the line of least resistance. The true teacher of mathematics will see growth and development in himself as well as in the pupils who sit in his class from day to day. It is a law of nature that life must spring from life, and surely we want that our pupils should drink from the living fountain rather than the stagnant pool. This means then, that the teacher of algebra, or geometry or trigonometry, should know more mathematics than just the subjects he is expected to teach. If he has studied the calculus, he knows that emphasis must be placed on factoring and the reduction of fractions in the algebra, and on remembering of trigonometric formulæ. If he would study function theory for a time, he would be more considerate of his pupils when they come to him puzzled over things which seem insignificant and which he feels ought to be mastered with little effort. And so, in conclusion, let me urge that the only true advancement is that which springs from consecrated teachers who are willing to prove all things and above all, hold fast that which is good.

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